

12.2

Surface Area of Prisms and Cylinders

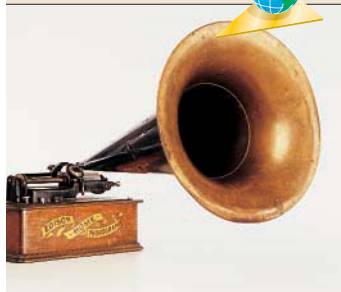
What you should learn

GOAL 1 Find the surface area of a prism.

GOAL 2 Find the surface area of a cylinder.

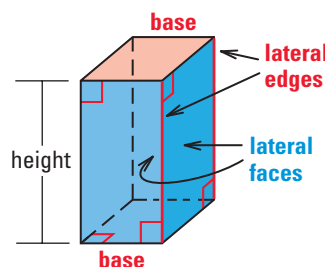
Why you should learn it

▼ You can find the surface area of **real-life** objects, such as the cylinder records used on phonographs during the late 1800s. See Ex. 43.

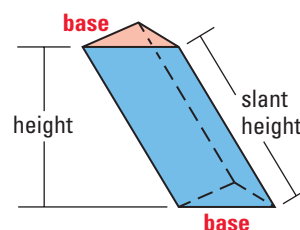
**GOAL 1** FINDING THE SURFACE AREA OF A PRISM

A **prism** is a polyhedron with two congruent faces, called **bases**, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are *lateral edges*.

The *altitude* or *height* of a prism is the perpendicular distance between its bases. In a **right prism**, each lateral edge is perpendicular to both bases. Prisms that have lateral edges that are not perpendicular to the bases are **oblique prisms**. The length of the oblique lateral edges is the *slant height* of the prism.



Right rectangular prism



Oblique triangular prism

Prisms are classified by the shapes of their bases. For example, the figures above show one rectangular prism and one triangular prism. The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

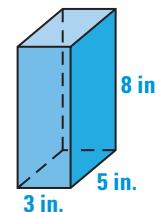
EXAMPLE 1 Finding the Surface Area of a Prism

Find the surface area of a right rectangular prism with a height of 8 inches, a length of 3 inches, and a width of 5 inches.

SOLUTION

Begin by sketching the prism, as shown. The prism has 6 faces, two of each of the following:

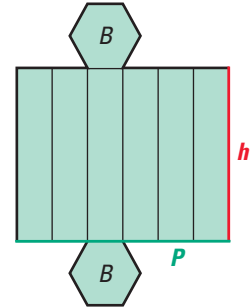
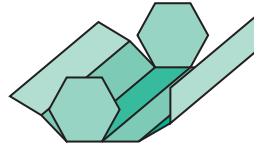
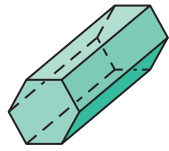
Faces	Dimensions	Area of faces
Left and right	8 in. by 5 in.	40 in.^2
Front and back	8 in. by 3 in.	24 in.^2
Top and bottom	3 in. by 5 in.	15 in.^2

**STUDENT HELP****Study Tip**

When sketching prisms, first draw the two bases. Then connect the corresponding vertices of the bases.

▶ The surface area of the prism is $S = 2(40) + 2(24) + 2(15) = 158 \text{ in.}^2$

Imagine that you cut some edges of a right hexagonal prism and unfolded it. The two-dimensional representation of all of the faces is called a **net**.



In the net of the prism, notice that the lateral area (the sum of the areas of the lateral faces) is equal to the perimeter of the base multiplied by the height.

THEOREM

THEOREM 12.2 Surface Area of a Right Prism

The surface area S of a right prism can be found using the formula $S = 2B + Ph$, where B is the area of a base, P is the perimeter of a base, and h is the height.

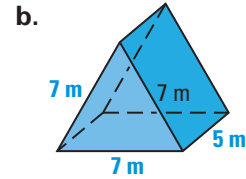
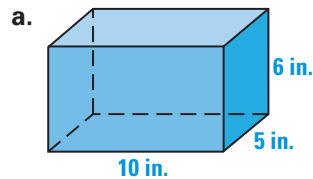
EXAMPLE 2 Using Theorem 12.2

STUDENT HELP

Study Tip

The prism in part (a) has three pairs of parallel, congruent faces. Any pair can be called bases, whereas the prism in part (b) has only one pair of parallel, congruent faces that can be bases.

Find the surface area of the right prism.



SOLUTION

- a. Each base measures 5 inches by 10 inches with an area of

$$B = 5(10) = 50 \text{ in.}^2$$

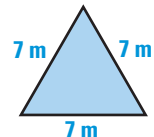
The perimeter of the base is $P = 30$ in. and the height is $h = 6$ in.

▶ So, the surface area is

$$S = 2B + Ph = 2(50) + 30(6) = 280 \text{ in.}^2$$

- b. Each base is an equilateral triangle with a side length, s , of 7 meters. Using the formula for the area of an equilateral triangle, the area of each base is

$$B = \frac{1}{4}\sqrt{3}(s^2) = \frac{1}{4}\sqrt{3}(7^2) = \frac{49}{4}\sqrt{3} \text{ m}^2.$$



The perimeter of each base is $P = 21$ m and the height is $h = 5$ m.

▶ So, the surface area is

$$S = 2B + Ph = 2\left(\frac{49}{4}\sqrt{3}\right) + 21(5) \approx 147 \text{ m}^2.$$

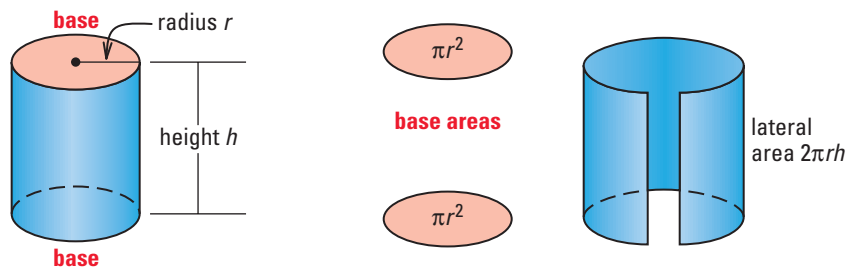
STUDENT HELP

Look Back

For help with finding the area of an equilateral triangle, see p. 669.

GOAL 2 FINDING THE SURFACE AREA OF A CYLINDER

A **cylinder** is a solid with congruent circular bases that lie in parallel planes. The *altitude*, or *height*, of a cylinder is the perpendicular distance between its bases. The radius of the base is also called the *radius* of the cylinder. A cylinder is called a **right cylinder** if the segment joining the centers of the bases is perpendicular to the bases.



The **lateral area of a cylinder** is the area of its curved surface. The lateral area is equal to the product of the circumference and the height, which is $2\pi rh$. The entire **surface area of a cylinder** is equal to the sum of the lateral area and the areas of the two bases.

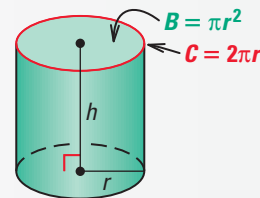
THEOREM

THEOREM 12.3 Surface Area of a Right Cylinder

The surface area S of a right cylinder is

$$S = 2B + Ch = 2\pi r^2 + 2\pi rh,$$

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.

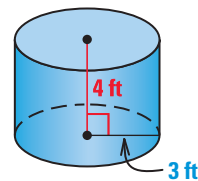


EXAMPLE 3 Finding the Surface Area of a Cylinder

Find the surface area of the right cylinder.

SOLUTION

Each base has a radius of 3 feet, and the cylinder has a height of 4 feet.



$$S = 2\pi r^2 + 2\pi rh$$

Formula for surface area of cylinder

$$= 2\pi(3^2) + 2\pi(3)(4)$$

Substitute.

$$= 18\pi + 24\pi$$

Simplify.

$$= 42\pi$$

Add.

$$\approx 131.95$$

Use a calculator.

► The surface area is about 132 square feet.

STUDENT HELP



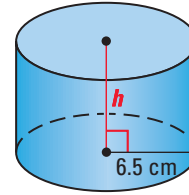
HOMEWORK HELP

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for extra examples.



EXAMPLE 4 Finding the Height of a Cylinder

Find the height of a cylinder which has a radius of 6.5 centimeters and a surface area of 592.19 square centimeters.



SOLUTION

Use the formula for the surface area of a cylinder and solve for the height h .

$$S = 2\pi r^2 + 2\pi rh$$

$$592.19 = 2\pi(6.5)^2 + 2\pi(6.5)h$$

$$592.19 = 84.5\pi + 13\pi h$$

$$592.19 - 84.5\pi = 13\pi h$$

$$326.73 \approx 13\pi h$$

$$8 \approx h$$

Formula for surface area

Substitute 6.5 for r .

Simplify.

Subtract 84.5π from each side.

Simplify.

Divide each side by 13π .

► The height is about 8 centimeters.

GUIDED PRACTICE

Vocabulary Check ✓

- Describe the differences between a prism and a cylinder. Describe their similarities.

Concept Check ✓

- Sketch a triangular prism. Then sketch a net of the triangular prism. Describe how to find its lateral area and surface area.

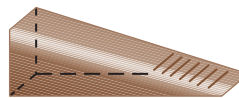
Skill Check ✓

Give the mathematical name of the solid.

- Soup can



- Door stop

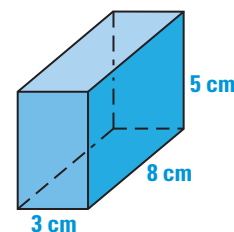


- Shoe box



Use the diagram to find the measurement of the right rectangular prism.

- Perimeter of a base
- Length of a lateral edge
- Lateral area of the prism
- Area of a base
- Surface area of the prism



Make a sketch of the described solid.

- Right rectangular prism with a 3.4 foot square base and a height of 5.9 feet
- Right cylinder with a diameter of 14 meters and a height of 22 meters

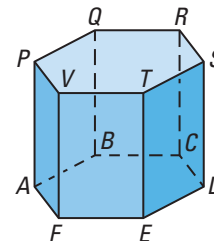
PRACTICE AND APPLICATIONS

STUDENT HELP

➔ **Extra Practice**
to help you master
skills is on p. 825.

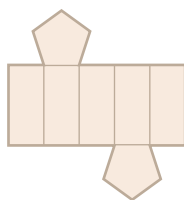
STUDYING PRISMS Use the diagram at the right.

13. Give the mathematical name of the solid.
14. How many lateral faces does the solid have?
15. What kind of figure is each lateral face?
16. Name four lateral edges.



ANALYZING NETS Name the solid that can be folded from the net.

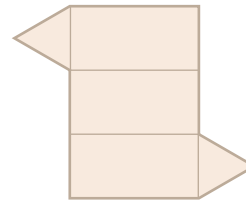
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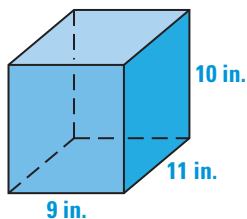


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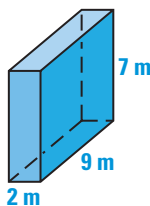


SURFACE AREA OF A PRISM Find the surface area of the right prism. Round your result to two decimal places.

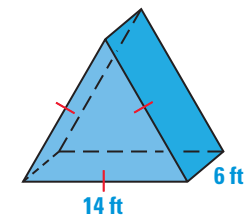
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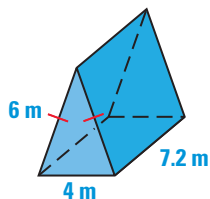
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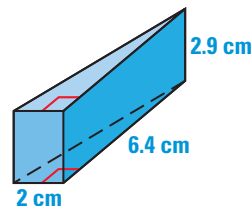
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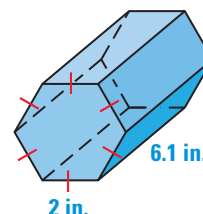
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24.

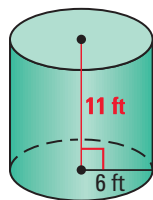


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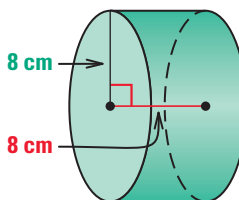


SURFACE AREA OF A CYLINDER Find the surface area of the right cylinder. Round the result to two decimal places.

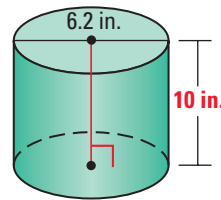
26.



27.



28.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 13–16,
20–25

Example 2: Exs. 20–25,
29–31, 35–37

Example 3: Exs. 26–28

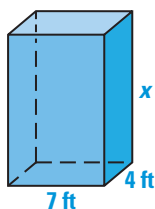
Example 4: Exs. 32–34

VISUAL THINKING Sketch the described solid and find its surface area.

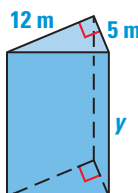
29. Right rectangular prism with a height of 10 feet, length of 3 feet, and width of 6 feet
30. Right regular hexagonal prism with all edges measuring 12 millimeters
31. Right cylinder with a diameter of 2.4 inches and a height of 6.1 inches

USING ALGEBRA Solve for the variable given the surface area S of the right prism or right cylinder. Round the result to two decimal places.

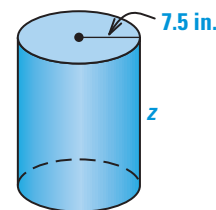
32. $S = 298 \text{ ft}^2$



33. $S = 870 \text{ m}^2$

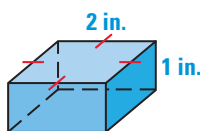


34. $S = 1202 \text{ in.}^2$

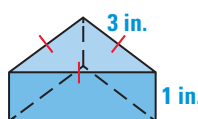


LOGICAL REASONING Find the surface area of the right prism when the height is 1 inch, and then when the height is 2 inches. When the height doubles, does the surface area double?

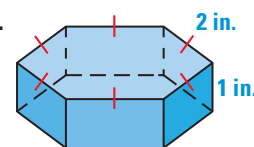
35.



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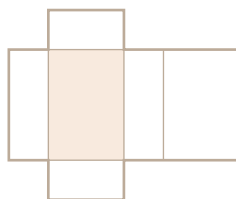


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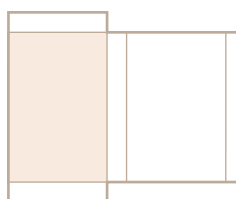


PACKAGING In Exercises 38–40, sketch the box that results after the net has been folded. Use the shaded face as a base.

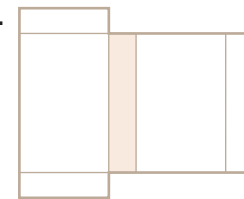
38.



39.



40.



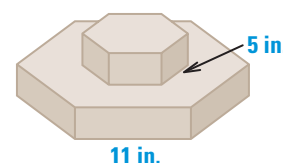
41. **CRITICAL THINKING** If you were to unfold a cardboard box, the cardboard would not match the net of the original solid. What sort of differences would there be? Why do these differences exist?

42. **ARCHITECTURE** A skyscraper is a rectangular prism with a height of 414 meters. The bases are squares with sides that are 64 meters. What is the surface area of the skyscraper (including both bases)?

43. **WAX CYLINDER RECORDS** The first versions of phonograph records were hollow wax cylinders. Grooves were cut into the lateral surface of the cylinder, and the cylinder was rotated on a phonograph to reproduce the sound. In the late 1800's, a standard sized cylinder was about 2 inches in diameter and 4 inches long. Find the exterior lateral area of the cylinder described.



44. **CAKE DESIGN** Two layers of a cake are right regular hexagonal prisms as shown in the diagram. Each layer is 3 inches high. Calculate the area of the cake that will be frosted. If one can of frosting will cover 130 square inches of cake, how many cans do you need? (Hint: The bottom of each layer will not be frosted and the entire top of the bottom layer will be frosted.)



FOCUS ON CAREERS

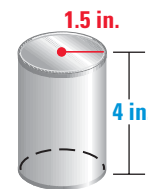


ARCHITECTS use the surface area of a building to help them calculate the amount of building materials needed to cover the outside of a building.

Test Preparation

MULTI-STEP PROBLEM Use the following information.

A canned goods company manufactures cylindrical cans resembling the one at the right.

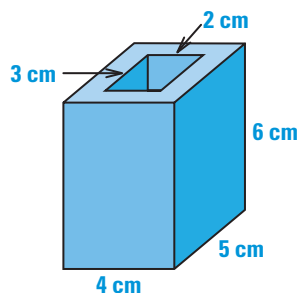


45. Find the surface area of the can.
46. Find the surface area of a can whose radius and height are twice that of the can shown.
47. *Writing* Use the formula for the surface area of a right cylinder to explain why the answer in Exercise 46 is not twice the answer in Exercise 45.

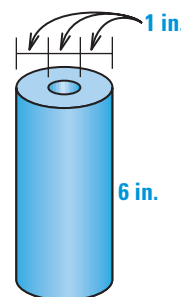
★ Challenge

FINDING SURFACE AREA Find the surface area of the solid. Remember to include both lateral areas. Round the result to two decimal places.

48.



49.



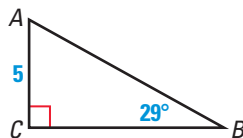
EXTRA CHALLENGE

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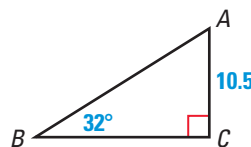
MIXED REVIEW

EVALUATING TRIANGLES Solve the right triangle. Round your answers to two decimal places. (Review 9.6)

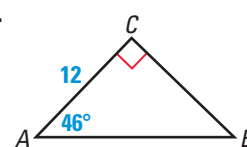
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51.

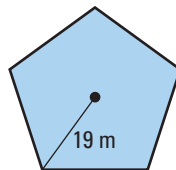


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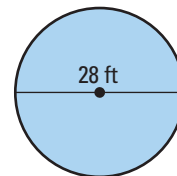


FINDING AREA Find the area of the regular polygon or circle. Round the result to two decimal places. (Review 11.2, 11.5 for 12.3)

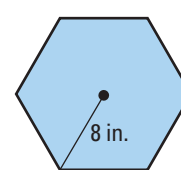
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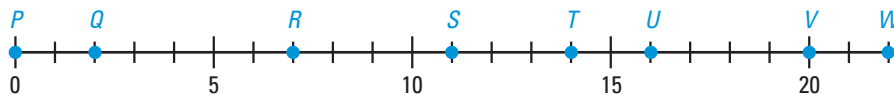
54.



55.



FINDING PROBABILITY Find the probability that a point chosen at random on \overline{PW} is on the given segment. (Review 11.6)

56. \overline{QS} 57. \overline{PU} 58. \overline{QU} 59. \overline{TW} 60. \overline{PV}